Earth Planets Space, **58**, 1213–1217, 2006

Kinetic effects on the parametric decays of Alfvén waves in relativistic pair plasmas

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(Received November 10, 2005; Revised April 13, 2006; Accepted April 17, 2006; Online published September 29, 2006)

Parametric decays of a circularly polarized wave propagating along a constant magnetic field in an electron-positron plasma are studied. Fully relativistic effects on the particle velocity in the wave field are considered, as well as kinetic effects in the parallel direction, by means of a one-dimensional relativistic Vlasov equation. In this approximation, a dispersion relation is found for the parametric decays which describes the coupling between normal modes of the system, namely electromagnetic sideband modes and Langmuir waves.
Key words: Parametric decays, Alfvén waves, relativistic plasma, electron-positron plasma.

1. Introduction

Electron-positron plasmas are different from electron-ion plasmas because, in the absence of a mass difference, there are no high or low natural frequency scales (Tsytovich and Wharton, 1978). Such plasmas are found in pulsar magnetospheres (Curtis, 1991; Ruderman and Sutherland, 1975; Cheng and Ruderman, 1980), models of the primitive Universe (Tajima and Taniuti, 1990), active galactic nuclei jets (Wardle *et al.*, 1998; Hirotani *et al.*, 2000; Wardle and Homan, 2001), and laboratory and tokamak plasmas (Zank and Greaves, 1995; Berezhiani and Mahajan, 1994). Relativistic effects are expected to play an important role in several of these systems. An understanding of the interactions between waves and relativistic electron-positron plasmas is relevant to proposed pulsar emission mechanisms (Luo and Melrose, 1992; Mahajan, 1997) and may provide insight into structure formation in the early Universe (Berezhiani and Mahajan, 1994).

The parametric instabilities of waves in unmagnetized electron-positron plasmas have been studied based on fluid theory (Gangadhara *et al.*, 1993; Shukla *et al.*, 1986; Gomberoff *et al.*, 1997; Muñoz and Gomberoff, 1998a), and on kinetic theory (Shukla and Stenflo, 2000). Some of these treatments include weakly relativistic effects on the particle motion in the wave field. However, in astrophysical environments, large particle energies are likely to be relevant. Consequently, fully relativistic effects have also been introduced (Muñoz and Gomberoff, 2002; Muñoz, 2004a,b), although, to date all these works deal with unmagnetized plasmas and, therefore, miss an important feature of space and astrophysical plasmas.

A dispersion relation for the parametric decays of electromagnetic waves in magnetized plasmas in electron-positron plasmas have been studied with fluid theory (Muñoz and Gomberoff, 1998b), in the weakly relativistic limit. In a later investigation Matsukiyo and Hada (2003) improved on that result by not assuming charge quasineutrality. They also performed full particle electromagnetic simulations and successfully compared their results with those resulting from fluid theory. However, in fluid theory important kinetic effects such as Landau damping are not present and, therefore, a kinetic theory is highly relevant to actually understanding the simulation results.

Thus, our aim in this work is to study the parametric decays of Alfvén waves in magnetized electron-positron plasmas, based on a kinetic theory and considering fully relativistic effects. In this paper, we calculate the dispersion relation for relativistic Alfvén waves propagating along a constant magnetic field in a pair plasma. Detailed numerical analysis of this dispersion relation and a comparison with particle simulations will be the subject of a future paper.

2. The Model

The basic equations are Maxwell equations and the collisionless Vlasov equation,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}_\perp \,, \tag{1}$$

$$\nabla^2\varphi = -4\pi e(n_p - n_e)\,, \tag{2}$$

$$0 = \frac{\partial f_j}{\partial t} + \vec{v}\cdot\vec{\nabla} f_j + q_j\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_{\vec{p}} f_j \,, \tag{3}$$

where $\vec{E} = -\vec{\nabla}\varphi$, $\vec{B} = \vec{\nabla}\times\vec{A}$ are the electric and magnetic fields, respectively, φ, $\vec{A}$ are the electrostatic and vector potentials, $\vec{J}_\perp$ is the total perpendicular current, and n_j and $f_j(\vec{r}, \vec{p}, t)$ are the density and distribution function for species j ($j = e$ for electrons, and $j = p$ for positrons).

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We assume that kinetic effects are only important along the parallel direction, so that we may approximate the distribution function in the perpendicular direction by the cold plasma expression,

$$f_j(z,\vec{p},t) = n_0 g_j(z,p_z,t)\delta\left(p_x - p_x^f\right)\delta\left(p_y - p_y^f\right) , \quad (4)$$

where g_j is the one-dimensional distribution function along the z direction, normalized to unity. n_0 represents the constant density of each species at equilibrium. $\vec{p}^f$ denotes the solution for the perpendicular momentum obtained from the fluid theory. This approximation is justified as long as energy transfer along the perpendicular direction can be neglected.

The density and perpendicular current density are then given by

$$n_j(z,t) = n_0 \int_{-\infty}^{\infty} dp_z\, g_j(z,p_z,t) , \quad (5)$$

$$\vec{J}_\perp(z,t) = n_0 \sum_j \int_{-\infty}^{\infty} dp_z \frac{\vec{p}_j^f}{\gamma_j} g_j , = n_0 \sum_j \int_{-\infty}^{\infty} dp_z\, \vec{v}_j^f g_j , \quad (6)$$

where

$$\gamma_j^2 = 1 + (p_z/mc)^2 + (p_j^f/mc)^2 . \quad (7)$$

Equations (1)–(3) can be written

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\frac{4\pi n_0}{c}\sum_j q_j \int_{-\infty}^{\infty} dp_z\, \vec{v}_j^f g_j , \quad (8)$$

$$\frac{\partial^2 \varphi}{\partial z^2} = -4\pi e n_0 \int_{-\infty}^{\infty} dp_z (g_p - g_e) , \quad (9)$$

$$0 = \frac{\partial g_j}{\partial t} + v_z \frac{\partial g_j}{\partial z} + q_j\left(-\frac{\partial \varphi}{\partial z} + \frac{1}{c}\vec{v}_j^f \times \vec{B}\right)\frac{\partial g_j}{\partial p_z} . \quad (10)$$

3. Fluid Theory for the Pump Wave

In Eq. (4) we need the particle momenta calculated from the fluid theory, which has been obtained by Matsukiyo and Hada (2003). For an Alfvén wave propagating along a constant magnetic field

$$\vec{B}_0 = B_{0z}\hat{z} ,$$

and for a wave of the form

$$\vec{A}_0 = A_0[\cos(k_0 z - \omega_0 t)\hat{x} + \sin(k_0 z - \omega_0 t)\hat{y}] , \quad (11)$$

particle momentum can be written

$$\vec{p}_j^f = -\frac{q_j}{c}\gamma_{j0}\xi_{j0}\vec{A}_0 , \quad (12)$$

where

$$\gamma_{j0} = \left(1 + \frac{p_j^{f\,2}}{m^2c^2}\right)^{1/2} , \quad \xi_{j0} = \frac{\omega_0}{\omega_0\gamma_{j0} - \Omega_j} , \quad (13)$$

and $\Omega_j = q_j B_{0z}/mc$ is the gyrofrequency of species j.

Also, the following dispersion relation is obtained:

$$\frac{c^2k_0^2}{\omega_0^2} = 1 - \sum_j \frac{\omega_{jp}^2}{\omega_0(\omega_0\gamma_{0j} - \Omega_j)} , \quad (14)$$

where $\omega_{jp} = (4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency of species j.

Although discussion of the dispersion relation of the pump wave is beyond the scope of this paper, we can outline the method of solution. Equations (12) and (13) yield the following polynomial equation for γ_{j0}:

$$\gamma_{j0}^4 + \gamma_{j0}^3\left(\mp\frac{2}{x}\right) + \gamma_{j0}^2\left(-1 + \frac{1}{x^2} - \frac{\eta^2}{y^2}\right) + \gamma_{j0}\left(\pm\frac{2}{x}\right) - \frac{1}{x^2} = 0 , \quad (15)$$

where the upper (lower) sign is for positrons (electrons), and $x = \omega_0/\Omega_p$, $y = ck_0/\Omega_p$, and $\eta = kA_0/B_{0z}$ is the ratio between the pump wave amplitude and the background magnetic field.

Thus, for a given wavenumber and pump wave amplitude, Eqs. (14) and (15) are a set of coupled equations which can be solved numerically for ω_0 and γ_{j0}. We can then proceed to study the parametric decays of the given wave.

4. Kinetic Theory for the Parametric Decays

We now consider Maxwell and Vlasov equations to study the parametric decays of the pump wave found in the previous section. We introduce the perpendicular quantities:

$$C_\perp = C_x + iC_y . \quad (16)$$

We perturb the electromagnetic field and g_j, assuming an equilibrium state consisting of a plasma composed by electrons and positrons, characterized by distribution functions $g_{j0}(p_z)$, and a circularly polarized electromagnetic wave of frequency ω_0 and wavenumber k_0, with potentials

$$\varphi_0 = 0 , \quad (17)$$

and $\vec{A}_0$ given by Eq. (11). We will also assume

$$g_{e0}(p_z) = g_{p0}(p_z) \equiv g_0(p_z) , \quad (18)$$

which is consistent with Eqs. (9) and (17). Perturbed quantities are given by

$$\delta g_j(p_z) = \tilde{g}_j(p_z)e^{i(kz-\omega t)} + \tilde{g}_j^*(p_z)e^{-i(k^*z-\omega^* t)} , \quad (19)$$

$$\delta\varphi(z,t) = \tilde{\varphi}e^{i(kz-\omega t)} + \tilde{\varphi}^* e^{-i(k^*z-\omega^* t)} , \quad (20)$$

$$\delta A_\perp = A_+ e^{i(k_+ z - \omega_+ t)} + A_- e^{i(k_- z - \omega_- t)} , \quad (21)$$

where

$$k_+ = k_0 + k , \quad \omega_+ = \omega_0 + \omega , \quad k_- = k_0 - k^* , \quad \omega_- = \omega_0 - \omega^* . \quad (22)$$

Then, to zeroth order Eqs. (8)–(10) yield:

$$0 = -\omega_0^2 + c^2 k_0^2 + \omega_p^2 \sum_j \int dp_z \, \frac{\omega_0}{\omega_0 \gamma_{j0} - \Omega_j} g_{j0} \,, \quad (23)$$

which generalizes the fluid dispersion relation (14).

On the other hand, to first order, Eqs. (8)–(10) yield:

$$\left(-k_+^2 + \frac{\omega_+^2}{c^2}\right) A_+ = -\frac{4\pi}{c} n_0 \sum_j q_j \times \int_{-\infty}^{\infty} dp_z (v_{j+} g_{j0} + v_{j0} \tilde{g}_j) \,, \quad (24)$$

$$\left(-k_-^2 + \frac{\omega_-^2}{c^2}\right) A_- = -\frac{4\pi}{c} n_0 \sum_j q_j \times \int_{-\infty}^{\infty} dp_z (v_{j-} g_{j0} + v_{j0} \tilde{g}_j^*) \,, \quad (25)$$

$$k^2 \tilde{\varphi} = 4\pi e n_0 \int_{-\infty}^{\infty} (\tilde{g}_p - \tilde{g}_e) \,, \quad (26)$$

$$0 = (-\omega + v_z k)\tilde{g}_j + q_j \times \left[-k\tilde{\varphi} - \frac{1}{2c}(-v_{j+} B_0 - v_{j0} B_-^* + v_{j-}^* B_0 + v_{j0} B_+)\right] \times \frac{\partial g_{j0}}{\partial p_z} \,. \quad (27)$$

The perpendicular velocities are obtained from the cold fluid equation of motion. Perturbing the fluid equations and using Eqs. (19)–(21), we obtain for the ω_+ Fourier component:

$$\tilde{v}_j k_0 p_j - \omega_+ = q_j \left[-\frac{\omega_+}{c k_+} B_+ + \frac{1}{c}(\tilde{v}_j B_0 - B_{z0} v_{j+})\right] \,. \quad (28)$$

In addition, from $\vec{p}_j = m\gamma_j \vec{v}_j$,

$$p_{j+} = m\gamma_{j0} \left[\frac{\gamma_{j0}^2}{2c^2} v_{j0}^2 v_{j-}^* + \left(\frac{\gamma_{j0}^2}{2c^2} v_{j0}^2 + 1\right) v_{j+}\right] \,. \quad (29)$$

Equations (28) and (29) yield

$$v_{j+}\left\{-\omega_+ \gamma_{j0}\left[\frac{1}{2}(\gamma_{j0}\xi_{j0} a_0)^2 + 1\right] + \Omega_j\right\} - \omega_+ \gamma_{j0} \frac{1}{2}(\gamma_{j0}\xi_{j0}a_0)^2 v_{j-}^* = \frac{q_j}{mc}\omega_+ A_+ - \tilde{v}_j \frac{q_j A}{mc} k_0 (1 - \gamma_{j0}\xi_{j0}) \,, \quad (30)$$

where we have defined the adimensional potential $\vec{a} = e\vec{A}/mc^2$.

A similar expression for v_{j-} is obtained, interchanging subscripts $+$ and $-$, and changing $\tilde{v}_j$ by $\tilde{v}_j^*$.

On the other hand, from the parallel cold fluid equation of motion we obtain:

$$\omega \gamma_{j0} \tilde{v}_j = \frac{q_j}{m}\left\{k\tilde{\varphi} + \frac{A_0}{2c}\left[k_0(v_{j+} - v_{j-}^*) - \frac{q_j \xi_{j0}}{mc}(k_-^* A_-^* - k_+ A_+)\right]\right\} \,. \quad (31)$$

Equations (24), (25), (26), (27), (30) (and the corresponding one for v_{j-}), and (31), are 11 algebraic equations for the 11 unknowns $A_\pm$, $\tilde{\varphi}$, $v_{j\pm}$, $\tilde{g}_j$, $\tilde{v}_j$, $j = p, e$. The kinetic dispersion relation is given by $\Delta = 0$, where Δ is the determinant of this system of equations.

The algebra is lengthy but straightforward, and leads to the following dispersion relation:

$$0 = K_{1+}(K_{2-}K_{30} - K_{3-}K_{20}) - K_{2+}(K_{1-}K_{30} - K_{3-}K_{10}) + K_{3+}(K_{1-}K_{20} - K_{2-}K_{10}) \,. \quad (32)$$

The coefficients K_{ab} are defined as follows:

$$K_{n\pm} = -\bar{k}_\pm^2 + \frac{\bar{\omega}_\pm^2}{c^2} + L_{n\pm} + \frac{\omega_p^2 m a_0}{2} \sum_j \sigma_j \int dp_z \, \frac{1}{\gamma_{j0}} \left(-\sigma_j \xi_{j0} a_0 + L_{n0}\frac{1}{k^2}\right) \times [k_0(-\tilde{E}_{j1\pm} + \tilde{E}_{j2\pm}) - \gamma_{j0}\xi_{j0}\bar{k}_\pm] \frac{\partial g_{j0}/\partial p_z}{\omega - k v_z} \,, \quad (33)$$

$$K_{n0} = \omega_p^2 m \sum_j \int dp_z \, \left(-\sigma_j \xi_{j0} a_0 + L_{n0}\frac{1}{k^2}\right) \times \left\{-k + \frac{k_0 \sigma_j a_0}{2\gamma_{j0}}(-\tilde{E}_{j10} + \tilde{E}_{j20})\right\} \frac{\partial g_{j0}/\partial p_z}{\omega - k v_z} \quad (34)$$

$$K_{3\pm} = \frac{\omega_p^2 m a_0}{2k^2} \sum_j \sigma_j \int dp_z \times \frac{1}{\gamma_{j0}}[k_0(-\tilde{E}_{j1\pm} + \tilde{E}_{j2\pm}) - \gamma_{j0}\xi_{j0}\bar{k}_\pm] \times \frac{\partial g_{j0}/\partial p_z}{\omega - k v_z} \quad (35)$$

$$K_{30} = \frac{\omega_p^2 m}{k^2} \sum_j \int dp_z \, \left\{-k + \frac{k_0 \sigma_j a_0}{2\gamma_{j0}}(-\tilde{E}_{j10} + \tilde{E}_{j20})\right\} \times \frac{\partial g_{j0}/\partial p_z}{\omega - k v_z} - 1 \quad (36)$$

$$L_{na} = \frac{\omega_p^2}{c^2} \tilde{F}_{na} \,, \quad (37)$$

$$\tilde{F}_{na} = \sum_j \int dp_z \, \frac{1}{\gamma_{j0}} \left\{\tilde{E}_{jna}\left[1 - \frac{1}{2}\left(\xi_{j0} a_0\right)^2\right] - \frac{1}{2}\left(\xi_{j0} a_0\right)^2 \tilde{E}_{jna}\right\} g_{j0} \,, \quad (38)$$

$$\tilde{E}_{j1+} = \frac{1}{C_j}\left[\bar{\omega}_+ \left(\left(\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}}\left(1 - \frac{1}{2}\xi_{j0}^2 a_0^2\right)\right)\omega + \frac{1}{2}\left(\frac{c a_0 k_0}{\gamma_{j0}}\right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right)\right) \quad (39)$$

$$- (c a_0)^2 \frac{1}{2} \xi_{j0} \bar{k}_+ \frac{k_0}{\gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}}\left(1 - \xi_{j0}^2 a_0^2\right)\right)\right] \,, \quad (40)$$

$$\tilde{E}_{j1-} = \frac{1}{C_j}\left[-\bar{\omega}_- \left(-\frac{\Omega_j}{\gamma_{j0}} \frac{1}{2}\xi_{j0}^2 a_0^2 \omega + \frac{1}{2}\left(\frac{c a_0 k_0}{\gamma_{j0}}\right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right)\right) \quad (41)$$

$$+ (ca_0)^2 \frac{1}{2} \xi_{j0} \bar{k}_- \frac{k_0}{\gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right) \Bigg] \quad (42)$$

$$\tilde{E}_{j2+} = \frac{1}{C_j} \left[-\bar{\omega}_+ \left(-\frac{\Omega_j}{\gamma_{j0}} \frac{1}{2} \xi_{j0}^2 a_0^2 \omega - \frac{1}{2} \left(\frac{c a_0 k_0}{\gamma_{j0}} \right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right) \right) \right. \quad (43)$$

$$\left. + (ca_0)^2 \frac{1}{2} \xi_{j0} \bar{k}_+ \frac{k_0}{\gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(-\bar{\omega}_+ + \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right) \right] \quad (44)$$

$$\tilde{E}_{j2-} = \frac{1}{C_j} \left[\bar{\omega}_- \left(\left(\bar{\omega}_+ - \frac{\Omega_j}{\gamma_{j0}} \left(1 - \frac{1}{2} \xi_{j0}^2 a_0^2 \right) \right) \omega - \frac{1}{2} \left(\frac{c a_0 k_0}{\gamma_{j0}} \right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right) \right) \right. \quad (45)$$

$$\left. - (ca_0)^2 \frac{1}{2} \xi_{j0} \bar{k}_- \frac{k_0}{\gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(-\bar{\omega}_+ + \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right) \right] , \quad (46)$$

$$\tilde{E}_{j10} = -\frac{\sigma_j c^2 k k_0}{C_j \gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right) , \quad (47)$$

$$\tilde{E}_{j20} = \frac{\sigma_j c^2 k k_0}{C_j \gamma_{j0}} \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left(-\bar{\omega}_+ + \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right) , \quad (48)$$

$$C_j = \left[-\bar{\omega}_+ + \frac{\Omega_j}{\gamma_{j0}} \left(1 - \frac{1}{2} \xi_{j0}^2 a_0^2 \right) \right] \times \left[\left(\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}} \left(1 - \frac{1}{2} \xi_{j0}^2 a_0^2 \right) \right) \omega + \frac{1}{2} \left(\frac{c a_0 k_0}{\gamma_{j0}} \right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right) \right] \quad (49)$$

$$- \frac{\Omega_j}{\gamma_{j0}} \frac{1}{2} \xi_{j0}^2 a_0^2 \left[-\frac{\Omega_j}{\gamma_{j0}} \frac{1}{2} \xi_{j0}^2 a_0^2 \omega + \frac{1}{2} \left(\frac{c a_0 k_0}{\gamma_{j0}} \right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right) \right] \quad (50)$$

$$+ \frac{1}{2} \left(\frac{c a_0 k_0}{\gamma_{j0}} \right)^2 \left(1 - \gamma_{j0}\xi_{j0}\right) \times \left[\bar{\omega}_- - \frac{\Omega_j}{\gamma_{j0}} \left(1 - \xi_{j0}^2 a_0^2\right) \right] . \quad (51)$$

In these equations, $n = 1, 2$, $a = \pm, 0$, and $j = e, p$, and we have introduced $\bar{k}_\pm = k_0 \pm k$, $\bar{\omega}_\pm = \omega_0 \pm \omega$.

In the absence of the pump wave ($A_0 = 0$), all coefficients K_{ab} in Eqs. (33)–(36) are zero, except

$$K_{1+} = -c^2 D_+ , \quad (52)$$

$$K_{2-} = -c^2 D_- , \quad (53)$$

$$K_{30} = -\epsilon , \quad (54)$$

where

$$D_+ = -\omega_+^2 + c^2 k_+^2 + \omega_p^2 \sum_j \int dp_z \, \frac{\omega_+}{\omega_+ \gamma_{j0} - \Omega_j} g_{j0} , \quad (55)$$

$$D_- = -\omega_-^{*\,2} + c^2 k_-^{*\,2} + \omega_p^2 \sum_j \int dp_z \, \frac{\omega_-^*}{\omega_-^* \gamma_{j0} - \Omega_j} g_{j0} , \quad (56)$$

$$\epsilon = 1 - \omega_p^2 \frac{m}{k} \sum_j \int dp_z \, \frac{\partial g_{j0}}{\partial p_z} \frac{1}{k v_z - \omega} , \quad (57)$$

are the dispersion relations of the electromagnetic sideband waves and the Langmuir waves.

Equation (32) then yields

$$0 = \epsilon D_+ D_- . \quad (58)$$

Thus, in the absence of the pump wave these are the possible wave modes of the system. For $A_0 \neq 0$, these normal modes couple, leading to the parametric decays of the system.

On the other hand, in the unmagnetized case $B_{0z} = 0$ we have

$$\Omega_j = 0 , \qquad \xi_{j0} = \frac{1}{\gamma_{j0}} , \qquad \gamma_{p0} = \gamma_{e0} \equiv \gamma_0 . \quad (59)$$

If we further assume

$$g_{p0} = g_{e0} \equiv g_0 , \quad (60)$$

the dispersion relation can be written in the form:

$$0 = \epsilon \left[D_+^{(0)} D_-^{(0)} + (D_+^{(0)} + D_-^{(0)}) \omega_p^2 a_0^2 c^2 \left(mk I_3 - \frac{I_4}{c}^2 \right) \right] , \quad (61)$$

where

$$D_+^{(0)} = -\omega_+^2 + c^2 k_+^2 + 2\omega_p^2 \int_{-\infty}^{\infty} dp_z \, \frac{g_0}{\gamma_0} , \quad (62)$$

$$D_-^{(0)} = -\omega_-^{*\,2} + c^2 k_-^{*\,2} + 2\omega_p^2 \int_{-\infty}^{\infty} dp_z \, \frac{g_0}{\gamma_0} , \quad (63)$$

and

$$I_3 = \int_{-\infty}^{\infty} dp_z \, \frac{1}{\gamma_0^2} \frac{\partial g_0 / \partial p_z}{v_z k - \omega} , \quad (64)$$

$$I_4 = \int_{-\infty}^{\infty} dp_z \frac{g_0}{\gamma_0^3} . \quad (65)$$

Equation (61) is the same result previously obtained by Muñoz (2004b) and Stenflo and Shukla (2004).

5. Nonrelativistic Temperature Limit

Solving the dispersion relation in the fully relativistic regime is complicated by the existence of branching points due to the γ_{j0} factors in the integrals. A simpler task is to consider the limit of nonrelativistic temperatures. In this

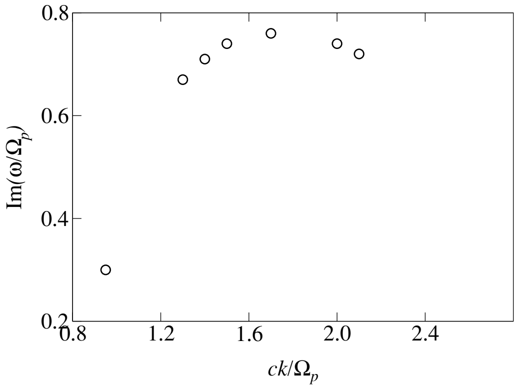

Fig. 1. Imaginary part of the frequency versus wavenumber for a parameter region where an instability develops. $v_{\rm th}/c = 0.1$, $ck_0/\Omega_p = 2$, $\omega = 0.7812$, $\eta = 0.1$, $\omega_p/\Omega_p = 1$.

case, g_0 can be taken to be a Maxwellian, and contributions from large p_z can be neglected in γ_{j0}. This greatly simplifies Eqs. (33)–(36). For

$$g_{j0}(p_z) = \frac{1}{m v_{\rm th}\pi} e^{-v_z^2/v_{\rm th}^2} , \qquad v_{\rm th} = \sqrt{\frac{2k_B T}{m}} , \qquad (66)$$

coefficients in Eq. (32) can be written in terms of the plasma dispersion function, using that

$$\int dp_z \frac{\partial g_{j0}/\partial p_z}{\omega - k v_z} = -\frac{1}{k m v_{\rm th}^2} Z'\left(\frac{\omega}{k v_{\rm th}}\right) . \qquad (67)$$

In Fig. 1 a solution for Eq. (32) for nonrelativistic temperatures is shown. This is only a particular instability found in the system. A thorough discussion of instabilities will be published elsewhere.

6. Summary

In this paper, the parametric decays of a circularly polarized wave propagating along a constant magnetic field in an electron-positron plasma have been studied. The treatment has been based on the relativistic Vlasov equation, so that kinetic effects and fully relativistic effects on the particle velocity in the wave field are considered. Kinetic effects have been considered only in the parallel direction. The dispersion relation for the parametric decays of the pump wave has been found. In the absence of the pump wave the normal modes of the system are the sideband waves and the Langmuir waves. When the pump wave is present, these modes couple, leading to the parametric instabilities of the system (see, e.g., Matsukiyo and Hada, 2003). For an unmagnetized plasma, the result is compatible with previously published results (Muñoz, 2004b; Stenflo and Shukla, 2004).

This dispersion relation can be solved numerically. Work is in progress to solve it for non-relativistic temperatures, where the plasma is described by a Maxwellian distribution function, and compare the results with particle simulations. This numerical analysis will be the subject of a future paper.

Acknowledgments. One of us (V.M.) acknowledges a Postdoctoral Fellowship granted by JSPS (Japan). This work has been partially supported by FONDECYT Grant No. 1060830.

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